NBF-003-1012008 Seat No. ________

# B. Sc. (Sem. II) (CBCS) Examination

**April / May - 2017**

## Mathematics : Paper - 02 (A) Theory

*(Geometry, Calculus & Matrix Algebra)*

**Faculty Code : 003**
**Subject Code : 1012008**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**Instruction :** All questions are compulsory.

1 (A) Answer the following questions briefly : **4**

(1) Find the centre and radius of the sphere $|\bar{r}|^2 - 2\bar{r}.(1,1,1) - 1 = 0$

(2) Find the sphere for which $(1,1,0)$ and $(0,1,1)$ are the extremities as a diameter.

(3) Write the equation of cylinder whose axis is parallel to Y-axis and radius $r$.

(4) Define cylinder.

(B) Answer any **one** out of two : **2**

(1) Find the radius of the circle that is obtain as the intersection of the plane $x+2y+2z=15$ and the sphere $x^2+y^2+z^2-2y-4z-20=0$

(2) Find the two tangent planes to the sphere $x^2+y^2+z^2-6x-4y-2z+5=0$ Which are parallel to the plane $8x+y+4z=0$

(C) Answer any **one** out of two : **3**

(1) Find equation of right circular cylinder with radius 2 and axis $\frac{x-1}{2} = y-2 = \frac{z-3}{2}$.

(2) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact

(D) Answer any **one** out of two : **5**

(1) Obtain the equation of cylinder whose generator is parallel to $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ passing through guiding curve $ax^2+2hxy+by^2+2gx+2fy+c=0,\ z=0.$

(2) A plane intersects co-ordinate axis in A,B,C and passes through (a,b,c). Prove that the centre of the sphere passing through O,A,B,C is on $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Here O is the origin and it is distinct from A,B,C.

**2** (A) Answer the following questions briefly : **4**

(1) Define limit of two variables function.

(2) Evaluate $\lim_{x\to 0}\left\{\lim_{y\to 0}\frac{x^2+y^2}{x-y}\right\}$

(3) Write formula of partial derivative $\frac{\partial f}{\partial y}$

(4) If $u=y^x$ then find $\frac{\partial u}{\partial y}$

(B) Answer any **one** out of two : **2**

(1) Using definition prove that $\lim_{(x,y)\to(2,1)} xy=2$

(2) If $x^3+y^3+3xy=0$ then show that $\frac{dy}{dx}=\frac{-\left(x^2+y\right)}{x+y^2}$

(C) Answer any **one** out of two : **3**

(1) If $W=\frac{y}{z}+\frac{z}{x}+\frac{x}{y}$ then prove that $x\frac{\partial W}{\partial x}+y\frac{\partial W}{\partial y}+z\frac{\partial W}{\partial z}=0$

(2) If $u=\sin\left(\frac{x^2+y^2}{xy}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

(D) Answer any **one** out of two : **5**

(1) State and prove Euler's theorem for homogeneous function of two variables.

(2) If $u=\sin^{-1}(x-y), x=3t, y=4t^3$ then show that $\frac{du}{dt}=\frac{3}{\sqrt{1-t^2}}$

3 (A) Answer the following questions briefly : **4**

(1) Define Jacobian

(2) Define local maxima and minima

(3) If $x = p\cos\theta$ and $y = p\sin\theta$ then find $\frac{\partial(p,\theta)}{\partial(x,y)}$

(4) Define extreme point.

(B) Answer any **one** out of two : **2**

(1) Find minimum value of $f(x,y) = x^3 + y^3 - 3xy$

(2) If $f(x,y) = x^2 y - 3y$ then find approximate value of $f(5.2, 6.85)$

(C) Answer any **one** of two : **3**

(1) If $u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$ then show that $\frac{\partial(u,v)}{\partial(x,y)} = 0$

(2) If $u = x^2 - 2y, v = x + y + z, w = x - 2y + 3z$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 10\,x + 4$

(D) Answer any **one** out of two : **5**

(1) State and prove Taylor's theorem.

(2) Expand log xy in power of $x-1$ and $y-1$.

4 (A) Answer the following questions briefly : **4**

(1) Define orthogonal matrix

(2) Give an example of 3×3 Symmetric matrix

(3) Define Transpose of a matrix

(4) Define skew symmetric matrix

(B) Answer any **one** out of two : **2**

(1) If $AB = A$ and $BA = B$ then prove that $A$ is idempotent matrix.

(2) If $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$ then find $A - B$.

(C) Answer any **one** out of two : **3**

(1) In usual notation prove that $(AB)^T = B^T A^T$

(2) Show that the matrix multiplication is associative.

(D) Answer any **one** out of two : **5**

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.

(2) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

**5** (A) Answer the following questions briefly : **4**

(1) Define eigen value

(2) Find the eigen value of $\begin{pmatrix} 5 & 3 \\ -1 & 1 \end{pmatrix}$

(3) Define characteristic equation of a matrix

(4) State Cayley Hamilton Theorem.

(B) Answer any **one** out of two : **2**

(1) For matrix $\begin{pmatrix} 5 & 3 \\ -1 & 1 \end{pmatrix}$, find the eigen vectors with respect to eigen value 2.

(2) Show that the matrices $A$ and $A^T$ have same eigen values.

(C) Answer any **one** out of two : **3**

(1) Solve : $x+2y+3z=0, 2x+3y+z=0, 3x+y+2z=0$

(2) Show that equation $AX = O$ has a non zero solution iff $A$ is singular matrix

(D) Answer any **one** out of two : **5**

(1) State and prove Cayley-Hamilton Theorem.

(2) Find eigen value and eigen vectors of

matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$